Quadratic convergence of Newton-Raphson iterations

This is a simple example that uses Python and `sympy` to demonstrate the quadratic converegnce of Newton-Raphson iterations to the exact root of a non-linear equation.

```
from sympy import *

x = Symbol('x')

f    = Lambda (x,x-exp(-x))
df   = Lambda (x,diff(f(x),x))
Step = Lambda (x,x-f(x)/df(x))

Digits = 200  # use 200 decimal digits for all numerical computations

x_new = Float('0.5',Digits)
f_new = N (f(x_new),Digits)

# pyBeg (table)

print ('\RuleA {:2d} & {: .25f} & {: .10e} &\\\\'.format(0,x_new,f_new))

for n in range (1,7):
   x_old = x_new
   x_new = N (Step(x_new),Digits)
   f_old = N (f(x_old),Digits)
   f_new = N (f(x_new),Digits)
   ratio = N (f_new / f_old**2,Digits)
   print ('\RuleA {:2d} & {: .25f} & {: .10e} & {: .5f}\\\\'.format(n,x_new,f_new,ratio))

# pyEnd (table)
```

Note the clear quadratic convergence in the iterations – the last column settles to approximately -0.11546 independent of the number of iterations. This behaviour would not be seen using normal floating point computations as they are normally limited to no more than 18 decimal digits. This computation used 200 decimal digits.

Newton-Raphson iterations $x_{n+1} = x_n - f_n/f'_n\ ,\quad f(x) = x - e^{-x}$			
n	x_n	$\epsilon_n = x_n - e^{-x_n}$	$\epsilon_n/\epsilon^2_{n-1}$
0	0.50000000000000000000000000	-1.0653065971e-1	
1	0.56631100319721815304164 92	-1.3045098060e-3	-0.11495
2	0.56714316503486221278651 21	-1.9648047172e-7	-0.11546
3	0.56714329040978102869957 66	-4.4574262753e-15	-0.11546
4	0.56714329040978387299996 87	-2.2941072910e-30	-0.11546
5	0.56714329040978387299996 87	-6.0767705445e-61	-0.11546
6	0.56714329040978387299996 87	-4.2637434326e-122	-0.11546

```
\def\RuleA{\vrule depth0pt  width0pt height14pt}
\def\RuleB{\vrule depth8pt  width0pt height14pt}
\def\RuleC{\vrule depth10pt width0pt height16pt}

\setlength{\tabcolsep}{0.025\textwidth}%

\begin{center}
   \begin{tabular}{cccc}%
      \noalign{\hrule height 1pt}
      \multicolumn{4}{c}{\RuleC\rmfamily\bfseries%
      Newton-Raphson iterations \quad%
      $x_{n+1} = x_n - f_n/f'_n\ ,\quad f(x) = x-e^{-x}$}\\
      \noalign{\hrule height 1pt}
      \RuleB$ n$&$ x_n$&$ \epsilon_{n} =  x_{n} - e^{-x_{n}}$&$\epsilon_{n}/\epsilon_{n-1}^2$\\
      \noalign{\hrule height 0.5pt}
      \py{table}
      \noalign{\hrule height 1pt}
   \end{tabular}
\end{center}
```